Chapter Twelve.

Simultaneous linear equations.

Introducing two variables.
The following example, and the solution that follows, appeared in chapter 7 where it was used to show how the introduction of an x allowed an equation to be built up and solved:

Example 1
An amateur drama group hire a theatre for their production. They expect to sell all of the 1 200 tickets, some at \$10 and the rest at \$7. The group require the ticket sales to cover their \$4150 production costs, to allow a donation of \$4000 to be made to charity and to provide a profit of \$1000 to aid future productions. If they are to exactly achieve this target, and their expectations regarding ticket sales are correct, how many of the total 1 200 tickets should they charge \$10 for and how many should they charge \$7 for?

Solution:

Let the number of \$7 tickets be	x		
These will give an income of	$7x$	dollars	
The number of \$10 tickets will then be	$(1\,200 - x)$		
These will give an income of	$10(1\,200 - x)$	dollars	
Thus	$7x + 10(1\,200 - x)$	$=$	$4150 + 4000 + 1000$
which can be simplified to	$12\,000 - 3x$	$=$	9150
Solving gives	x	$=$	950

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.

Instead of introducing a single variable, x, for the number of \$7 tickets, we could introduce **two variables**, x and y, where x is the number of \$7 tickets and y is the number of \$10 tickets:

They expect to sell 1 200 tickets	$\therefore$	$x + y$	$=$	$1\,200$	①
The total revenue must be \$9 150 tickets	$\therefore$	$7x + 10y$	$=$	$9\,150$	②

We now have **two equations** each involving the same two unknowns, x and y.

Two equations involving the same two variables can be solved together, *simultaneously*, to determine the two variables.
These "simultaneous equations" as they are called can be solved by:

- using the simultaneous equation solving capability of some calculators,
- using the two equations "against each other" to reduce to just <u>one</u> equation involving <u>one</u> variable, which can then be determined.
- using a graphical approach.

The following example is a repeat of the example on the previous page but the solution given below uses <u>two</u> variables and demonstrates the three methods of solution mentioned on the previous page.

Example 2 (Example 1 re-visited).
An amateur drama group hire a theatre for their production. They expect to sell all of the 1200 tickets, some at \$10 and the rest at \$7. The group require the ticket sales to cover their \$4150 production costs, to allow a donation of \$4000 to be made to charity and to provide a profit of \$1000 to aid future productions. If they are to exactly achieve this target, and their expectations regarding ticket sales are correct, how many of the total 1 200 tickets should they charge \$10 for and how many should they charge \$7 for?

Let the number of \$7 tickets be x
and the number of \$10 be y

They expect to sell 1 200 tickets $\quad \therefore \quad x + y = 1200 \quad$ ①

The total revenue must be \$9 150 tickets $\quad \therefore \quad 7x + 10y = 9150 \quad$ ②

1. Solve using the equation solving capability of some calculators.

The display on the right shows the two equations put into a calculator and the values

$x = 950$

and $y = 250$ displayed.

These are the values that "fit" both the equation $x + y = 1200$ <u>and</u> $7x + 10y = 9150$.

Check: $1(950) + 1(250) = 1\,200$ ✓

and $7(950) + 10(250) = 9\,150$ ✓

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.

$$\left\{\begin{array}{l} x + y = 1200 \\ 7x + 10y = 9150 \end{array}\right| x, y$$

$\{x{=}950,\ y{=}250\}$

Some calculators require you to input the equations in a different form.
The display below left for example shows the equations put into the calculator by entering:

by entering:	1	1	1 200	for	$\mathbf{1x + 1y = 1200}$
and	7	10	9 150	for	$\mathbf{7x + 10y = 9150}$

Below right shows the pair of values that satisfy these two equations.

anX+bnY=Cn

$$\begin{array}{cccc} & a & b & c \\ \begin{array}{c}1\\2\end{array} & \multicolumn{3}{c}{\begin{bmatrix} 1 & 1 & 1200 \\ 7 & 10 & 9150 \end{bmatrix}} \end{array} \Rightarrow$$

anX+bnY=Cn

$$\begin{array}{c} X \\ Y \end{array} \begin{bmatrix} 950 \\ 250 \end{bmatrix}$$

As before, the values that fit both equations are $x = 950$ and $y = 250$.

Note: In the equation $7x + 10y = 9\,150$, 7 and 10 are the **coefficients** of x and y.

2. **Solve using the two equations "against each other" to reduce to just <u>one</u> equation involving <u>one</u> variable.**

Method 1, Substitution.

We have the two equations:	$x + y = 1\,200$	①
	$7x + 10y = 9\,150$	②
From equation ① we obtain y in terms of x	$y = 1\,200 - x$	
Substitute this expression for y into ②	$7x + 10(1\,200 - x) = 9\,150$	
Thus	$7x + 12\,000 - 10x = 9\,150$	
i.e.	$12\,000 - 3x = 9\,150$	
Add $3x$ to both sides to make the x term positive:	$12\,000 = 9\,150 + 3x$	
Subtract 9 150 from both sides to isolate $3x$:	$2\,850 = 3x$	
Divide both sides by 3 to isolate x:	$950 = x$	
i.e.	$x = 950$	
From $y = 1\,200 - x$ it then follows that	$y = 1\,200 - 950$	
	$= 250$	

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.

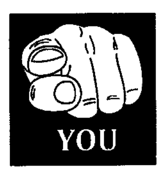
Method 2, Elimination.

In this method the strategy is to manipulate the equations until the coefficient of one of the variables is the same (except possibly for their sign) in both equations, and then to either add or subtract the two equations to eliminate that variable.

We have the two equations:	$x + y = 1\,200$			①
	$7x + 10y = 9\,150$			②
× ① by 10 so that it features $10y$:	$x + y = 1\,200$	×10→	$10x + 10y = 12\,000$	③
Keep ② unchanged	$7x + 10y = 9\,150$	→	$7x + 10y = 9\,150$	②
Equation ③ – equation ② :			$3x = 2\,850$	
		∴	$x = 950$	
From $x + y = 1\,200$ it then follows that			$y = 250$	

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.

If you are required to be able to solve simultaneous equations without using the inbuilt features of some calculators, make sure you attempt some of the questions of the next exercise in that way.



3. **Solve using a graphical approach.**

The equation $x + y = 1200$ has many possible solutions, some of which are shown below:

x:	0	1	10	15	33	98	117	900	950	1116
y:	1200	1199	1190	1185	1167	1102	1083	300	250	84
$x + y$:	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200

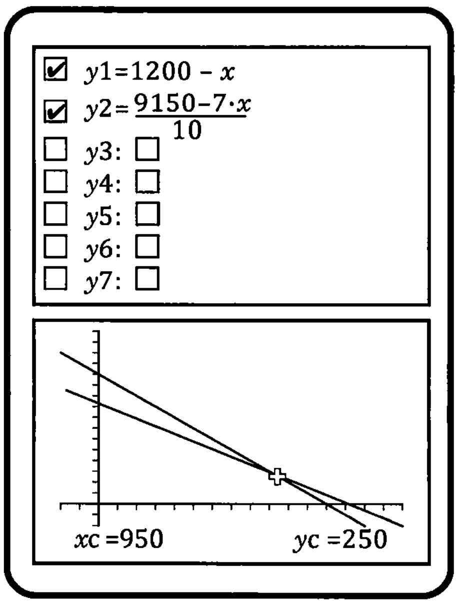
The equation $7x + 10y = 9150$ has many possible solutions, some of which are shown below:

x:	0	10	20	50	100	210	250	900	950	1120
y:	915	908	901	880	845	768	740	285	250	131
$7x + 10y$:	9150	9150	9150	9150	9150	9150	9150	9150	9150	9150

Notice that the above tables both include $x = 950$ and $y = 250$, the pair of values that fit both of the equations. Thus $x = 950$ and $y = 250$ are the solutions to the equations. However we could not be sure of being this lucky when we randomly list possible pairs for each of two equations. However we can use this idea to solve the equations graphically.

The graph of the equation $x + y = 1\,200$ will pass through all of the points whose x and y coordinates fit the equation $x + y = 1\,200$.

Similarly the graph of $7x + 10y = 9\,150$ will pass through all of the points whose x and y coordinates fit the equation $7x + 10y = 9\,150$.



Using a graphic calculator to plot both lines, the x and y coordinates of the point where the lines intersect will fit both equations (and of course with both equations being for straight lines there will be only one such point of intersection). The display on the right shows the equations, their graphs and the coordinates of the point of intersection.

Thus solving the equations $\begin{cases} x + y = 1\,200 \\ 7x + 10y = 9\,150 \end{cases}$

gives $x = 950$ and $y = 250$.

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.

- Some calculators require the equations to be input in the form "$y =$", as shown. Thus
  $x + y = 1\,200$ was entered as $y = 1200 - x$
  $7x + 10y = 9150$ was entered as $y = \dfrac{9150 - 7x}{10}$.
- You may need to adjust the viewing window of your calculator so that the point of intersection can be seen (though some calculators will state the coordinates of the point of intersection even if it is off screen).

Question: Which of these methods for solving simultaneous equations should you use?

Answer: Unless your teacher, or a particular question, requires you to use a particular method use whichever method appeals to you most for that question, and with which you would expect to make least mistakes. **However do make sure you know which methods you might be required to demonstrate in this unit and practice those.**

The method(s) you are likely to use need practice. The next two examples further demonstrate the technique of using two equations in two unknowns to give one equation in one unknown. This is not to suggest that this should be your chosen method for these questions but rather to further demonstrate its application and allow those readers who intend using other methods to check that they can obtain the same answers "their way".

Example 3 shows the substitution approach and example 4 the elimination approach.

Example 3

(a) Solve $\begin{cases} 3x = 5y - 10 \\ x + y = 34 \end{cases}$ (b) Solve $\begin{cases} 0{\cdot}3A + 0{\cdot}1P = 161 \\ 5A - 3P = 1050 \end{cases}$

(a)

	$3x = 5y - 10$ ①
	$x + y = 34$ ②
From ②	$y = 34 - x$
Substitute into ①	$3x = 5(34 - x) - 10$
Expand:	$3x = 170 - 5x - 10$
Add $5x$ to both sides:	$8x = 170 - 10$
i.e.	$8x = 160$
Divide both sides by 8	$x = 20$
But $y = 34 - x$, thus	$y = 34 - 20$
	$= 14$

Hence $x = 20$ and $y = 14$.

(b)

	$0{\cdot}3A + 0{\cdot}1P = 161$ ①
	$5A - 3P = 1050$ ②
× ① by 10 to avoid decimals:	$3A + P = 1610$
Thus	$P = 1610 - 3A$
Substitute into ②	$5A - 3(1610 - 3A) = 1050$
Expand:	$5A - 4830 + 9A = 1050$
i.e.	$14A - 4830 = 1050$
Add 4830 to both sides:	$14A = 5880$
Divide both sides by 14:	$A = 420$
But $P = 1610 - 3A$ and so	$P = 1610 - 3 \times 420$
and so	$P = 350$

Thus $A = 420$ and $P = 350$.

Example 4

(a) Solve $\begin{cases} 3x + 2y = 11 \\ x + 2y = 1 \end{cases}$ (b) Solve $\begin{cases} 5x - 2y = 6 \\ 3x + 2y = 26 \end{cases}$ (c) Solve $\begin{cases} 2x + 3y = 12 \\ x + 4y = 11 \end{cases}$

(a) *Notice that both equations feature "+ 2y". Taking one equation from the other will take "2y" from itself and thus eliminate one variable.*

$$\begin{array}{lrcl} & 3x + 2y &=& 11 \quad ① \\ & \underline{x + 2y} &\underline{=}& \underline{1} \quad ② \\ ① - ②: & 2x + 0y &=& 10 \\ \text{i.e.} & 2x &=& 10 \\ \text{Thus} & x &=& 5 \\ \text{Substitute } x = 5 \text{ into } ② & 5 + 2y &=& 1 \\ \text{Take 5 from both sides:} & 2y &=& -4 \\ \text{Hence:} & y &=& -2 \end{array}$$

Thus $x = 5$ and $y = -2$.

(b) *Notice that one equation features "– 2y" and the other features "+ 2y". Adding the equations together will allow these to eliminate each other.*

$$\begin{array}{lrcl} & 5x - 2y &=& 6 \quad ① \\ & \underline{3x + 2y} &\underline{=}& \underline{26} \quad ② \\ ① + ②: & 8x + 0y &=& 32 \\ \text{i.e.} & 8x &=& 32 \\ \text{Thus} & x &=& 4 \\ \text{Substitute } x = 4 \text{ into } ② & 12 + 2y &=& 26 \\ \text{Take 12 from both sides:} & 2y &=& 14 \\ \text{Hence:} & y &=& 7 \end{array}$$

Thus $x = 4$ and $y = 7$.

(c) *If we leave the first equation unchanged but multiply the second equation by 2 we will have two equations each featuring "2x". Taking one equation from the other will then eliminate one variable.*

$$\begin{array}{rcll} 2x + 3y &=& 12 \quad ① & \rightarrow \quad 2x + 3y = 12 \\ x + 4y &=& 11 \quad ② & \times 2 \rightarrow \quad \underline{2x + 8y = 22} \end{array}$$

$$\begin{array}{lrcl} \text{Subtracting:} & -5y &=& -10 \\ \text{Giving} & y &=& 2 \\ \text{From } ② & x + 4(2) &=& 11 \\ & x &=& 3 \end{array}$$

Thus $x = 3$ and $y = 2$.

Solving word problems.
In the next two examples the two equations must first be determined from the information given and then the equations can be solved. Notice that in each example the variables that are to be used are clearly introduced in the working.

Example 5
Two numbers have a difference of 8 whilst three times the larger added to twice the smaller totals 59. Find the two numbers.

Let the smaller number be x and the larger number be y.

The two numbers have a difference of 8 $\quad\therefore\quad y - x = 8 \quad$ ①
3 times the larger + 2 times the smaller = 59 $\quad\therefore\quad 3y + 2x = 59 \quad$ ②

Equations ① and ② can be solved to give $x = 7$
and $y = 15$

$$\left\{\begin{array}{l} y - x = 8 \\ 3y + 2x = 59 \end{array}\right| x, y$$

$$\{x=7,\ y=15\}$$

The two numbers are 7 and 15.

Example 6
Every one of the 4 000 tickets for a music concert at an entertainment centre is sold. Some of the tickets cost \$28 each and the remainder cost \$19 each. If the total revenue from the sale of the tickets is \$83 200 find how many of the 4 000 tickets cost \$28 and how many cost \$19.

Suppose x tickets cost \$28 and y tickets cost \$19.

There were 4 000 tickets altogether $\quad\therefore\quad x + y = 4\,000 \quad$ ①
The total revenue from the tickets sales was \$83 200 $\quad\therefore\quad 28x + 19y = 83\,200 \quad$ ②

Equations ① and ② can be solved to give $x = 800$
and $y = 3\,200$

Thus 800 of the tickets cost \$28 and 3 200 of the tickets cost \$19.

Notice that each of the two examples above finish with a clear statement of the answer. Example 5 does not end with $x = 7$ and $y = 15$. The question posed had no mention of x and y – we chose to introduce them to help us to solve the problem. The final answer should be expressed in the context of the question.
Hence example 5 ends with: *The two numbers are 7 and 15* and example 6 with *Thus 800 of the tickets cost \$28 and 3 200 of the tickets cost \$19.*

Note: It is <u>not</u> the intention here to claim that these questions can only be solved by introducing two letters, building up two equations and solving them simultaneously. Questions like the previous example can be solved by introducing just one variable, as we saw at the start of this chapter.

Alternatively the solution could be "reasoned through" as follows:

Selling all 4000 at \$19 would have raised $\quad 4000 \times \$19 = \$76\,000$

However the ticket sales raised \$83 200, i.e. \$7 200 "extra". This extra must come from the extra \$9 charged on some tickets.

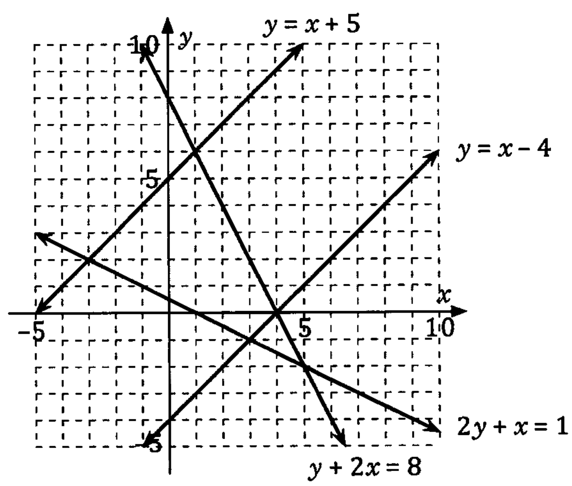
\$7 200 is 800 lots of \$9 so 800 seats were priced at \$28 and 3200 at \$19.

Yet another method would be to *guess* the number of \$28 tickets there should be, *check* whether our guess works and then *improve* our guess, i.e. *guess, check and improve.*

Introducing two letters and solving the resulting pair of equations simultaneously can be very useful but other methods can be just as effective. In general, for each question you should choose the method you consider most appropriate for you to use to solve that question. However do use the next exercise to practice the techniques shown in this chapter.

Exercise 12A.

Use the graph shown on the right to solve the following pairs of equations simultaneously.



1. $y = x + 5$ and $y + 2x = 8$
2. $y = x - 4$ and $2y + x = 1$
3. $y = x - 4$ and $y + 2x = 8$
4. $y + 2x = 8$ and $2y + x = 1$
5. $y = x + 5$ and $2y + x = 1$

Solve the following pairs of equations.

6. $\begin{cases} 2x + y = 19 \\ x - y = 2 \end{cases}$

7. $\begin{cases} 3x + 2y = 17 \\ x + 2y = 11 \end{cases}$

8. $\begin{cases} 5x + 3y = 9 \\ 5x - y = 17 \end{cases}$

9. $\begin{cases} 3x + 2y = -1 \\ -x + 2y = 27 \end{cases}$

10. $\begin{cases} 3x - y = 16 \\ 2x + y = 29 \end{cases}$

11. $\begin{cases} 2x - 3y = 16 \\ x - 3y = 11 \end{cases}$

12. $\begin{cases} 3x + 5y = 47 \\ 3x + 2y = 26 \end{cases}$

13. $\begin{cases} -x + 7y = 3 \\ x - 3y = 1 \end{cases}$

14. $\begin{cases} 2x + 3y = 12 \\ 2x - y = -12 \end{cases}$

15. $\begin{cases} y = x - 3 \\ 2x + 3y = 11 \end{cases}$
16. $\begin{cases} 2x + y = 7 \\ 3x - 2y = 14 \end{cases}$
17. $\begin{cases} 3x - 2y = 6 \\ 2x - 3y = -1 \end{cases}$

18. $\begin{cases} y = 11 - 2x \\ 2x + 3y = 21 \end{cases}$
19. $\begin{cases} 3x - 5y = 6 \\ x = 2y + 1 \end{cases}$
20. $\begin{cases} 3A + 2B = 11 \\ 3A - 2B = 19 \end{cases}$

21. $\begin{cases} 2p - 3q = 2 \\ 4p + 2q + 1 = 29 \end{cases}$
22. $\begin{cases} 0{\cdot}5x + 0{\cdot}2y = 7 \\ 2x - 3y = -29 \end{cases}$
23. $\begin{cases} 2(x + 5) = 3y \\ x + 2y = 30 \end{cases}$

24. One day a baker bakes x white loaves and y wholemeal loaves.
   (a) The number of white loaves baked that day together with the number of wholemeal loaves baked that day totalled 600.
   Which of the following equations correctly expresses this information?

Equation 1	Equation 2	Equation 3
$x - y = 600$	$y - x = 600$	$x + y = 600$

   (b) The number of white loaves baked that day exceeded the number of wholemeal loaves baked that day by 140.
   Which of the following equations correctly expresses this information?

Equation 4	Equation 5	Equation 6
$x - y = 140$	$y - x = 140$	$x + y = 140$

   (c) Given that both of the statements from (a) and (b) are correct solve your equations from parts (a) and (b) to determine the number of each type of loaf the baker baked that day.

25. At a dog show there were x people (each with two legs) and y dogs (each with four legs).
   (a) The total number of legs at the show that were either human legs or dog legs equalled 1758.
   Which of the following equations correctly expresses this information?

Equation 1	Equation 2	Equation 3
$2x + 4y = 1758$	$4x + 2y = 1758$	$x + y = 1758$

   (b) If the number of dogs at the show is multiplied by 5 and the answer subtracted from the number of people at the show the number obtained is 403.
   Which of the following equations correctly expresses this information?

Equation 4	Equation 5	Equation 6
$5x - y = 403$	$x - 5y = 403$	$5y - x = 403$

   (c) Given that both of the statements from (a) and (b) are correct solve your equations from parts (a) and (b) to determine the number obtained by adding the number of people at the show to the number of dogs at the show.

26. The smaller of two numbers is x and the larger is y.
   (a) The larger of the two numbers is 5 more than the smaller.
   Which of the following equations correctly expresses this information?

Equation 1	Equation 2	Equation 3
$x - y = 5$	$y - x = 5$	$x + y = 5$

   (b) Twice the smaller added to three times the larger equals 70.
   Which of the following equations correctly expresses this information?

Equation 4	Equation 5	Equation 6
$2x + 3y = 70$	$3x + 2y = 70$	$x + y = 70$

   (c) Given that both of the statements from (a) and (b) are correct solve your equations from parts (a) and (b) to determine the two numbers.

27. Sally saves \$1 and 50 cent coins by putting them into a piggy bank. When Sally opens the piggy bank she finds that it contains x \$1 coins and y 50 cent coins.
   (a) The piggy bank contained 46 coins altogether, all either \$1 coins or 50 cent coins. Which of the following correctly expresses this information?

Equation 1	Equation 2	Equation 3
$x + y = 46$	$xy = 46$	$x + 0{\cdot}5y = 46$

   (b) The total value of the coins in the piggy bank came to \$32.
   Which of the following correctly expresses this information?

Equation 4	Equation 5	Equation 6
$2x + y = 32$	$x + 0{\cdot}5y = 32$	$0{\cdot}5x + y = 32$

   (c) Solve your equations from parts (a) and (b) to determine how many of each type of coin the piggy bank contained.

28. A seamstress buys:
   x metres of material A, costing \$28 per metre,
   and y metres of material B, costing \$35 per metre.
   (a) The seamstress buys a total of 23 metres of these two materials.
   Which of the following correctly expresses this information?

Equation 1	Equation 2	Equation 3
$x + y = 28$	$x + y = 35$	$x + y = 23$

   (b) The two quantities cost the seamstress a total of \$700 altogether.
   Which of the following correctly expresses this information?

Equation 4	Equation 5	Equation 6
$35x + 28y = 700$	$xy + 28 + 35 = 700$	$28x + 35y = 700$

   (c) Solve your equations from parts (a) and (b) to determine what length of each material the seamstress bought.

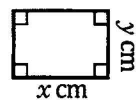
29. An investor invests $\$x$ in company X and $\$y$ in company Y.
   (a) The investor invests a total of \$25 000 in these two companies.
   Which of the following correctly expresses this information?

Equation 1	Equation 2	Equation 3
$X + Y = 25\,000$	$x + y = \$25\,000$	$x + y = 25\,000$

   (b) The investment in company X achieves a 4% loss in value and the investment in company Y achieves a 12% increase in value to make the total investment now worth \$25 120.
   Which of the following correctly expresses this information?

Equation 4	Equation 5	Equation 6
$-4x + 12y = 25\,120$	$0{\cdot}04x + 0{\cdot}12y = 25\,120$	$0{\cdot}96x + 1{\cdot}12y = 25\,120$

   (c) Solve your equations from parts (a) and (b) to determine the amount the investor put into each company.

30. A rectangle has a length of x cm and a height of y cm.


   (a) The perimeter of the rectangle is 70 cm.
   Which of the following correctly expresses this information?

Equation 1	Equation 2	Equation 3
$x + y = 35$	$xy = 70$	$x + y = 70$

   (b) Three heights exceeds two lengths by 15 cm.
   Which of the following correctly expresses this information?

Equation 4	Equation 5	Equation 6
$3x - 2y = 15$	$3y - 2x = 15$	$3x + 2y = 15$

   (c) Solve your equations from parts (a) and (b) to determine x and y and hence determine the area of the rectangle.

31. Entry into a particular event costs $\$x$ for each adult and $\$y$ for each child.
   (a) For 16 adults and 7 children the total cost is \$256.
   Which of the following equations correctly expresses this information?

Equation 1	Equation 2	Equation 3
$16x + 7y = 265$	$16y + 7x = 256$	$16x + 7y = 256$

   (b) For 20 adults and 11 children the total cost is \$338.
   Which of the following equations correctly expresses this information?

Equation 4	Equation 5	Equation 6
$20y + 11x = 338$	$20x + 11y = \$338$	$20x + 11y = 338$

   (c) Solve your equations from parts (a) and (b) to determine the total cost of entry for five adults and three children.

32. Two numbers have a sum of 41 whilst three times the larger added to twice the smaller totals 106. By letting the smaller number be x and the larger be y express the given information as two equations and hence determine the two numbers.

33. Two numbers have a difference of eleven whilst five times the smaller exceeds twice the larger by seventeen. Find the two numbers.

34. A chemist is asked to make 100 mL of a particular medicine. This 100 mL should contain 20 g of a certain compound. The chemist has available two bottles of this medicine but neither is to the desired concentration. The solution in bottle A has 0·15g per mL and the solution in bottle B has 0·4 g per mL. How many mL should the chemist use from each bottle to make the 100 mL of the desired concentration? (Hint: Let the required mix consist of x mL from A and y mL from B.)

35. A person has \$12 000 to invest in two companies, Acorp and Bcorp. The person invests \$$x$ with Acorp and \$$y$ with Bcorp. After one year each \$1 invested with Acorp has grown to \$1·12 and each \$1 invested with Bcorp has grown to \$1·05. The person's \$12 000 has grown to \$13 195.
    (a) Write two equations involving x and y.
    (b) Solve these equations to determine x and y.

36. The total amount received from the sale of 1500 tickets for a play is \$13 800. Some of the tickets were sold for \$12 each and the rest for \$8 each. How many tickets were sold for \$12 and how many for \$8?

37. A coach hire company has 25 coaches. Some of these can carry 56 passengers each and the rest 35 passengers each. With all 25 coaches full 1211 passengers can be carried. How many of each size coach does the company have?

38. Two numbers are such that seven times the smaller exceeds three times the larger by one whilst twice the larger exceeds four times the smaller by four. Find the two numbers.

39. A stall at a school fete sold jars of jam, for \$2·50 per jar, and jars of relish, for \$2 per jar. In all they sold 78 jars of these two commodities, receiving a total of \$179. How many jars of each commodity did they sell?

40. To start a new company a person borrows \$120 000 from a bank. Under the terms of the loan the company will pay interest on this loan calculated at 14% per annum on part of the loan and 17% per annum on the remainder and does not have to repay any of the \$120 000 capital until the second year. If the first year interest bill totalled \$18 150 how much of the \$120 000 was borrowed at 14% and how much at 17%?

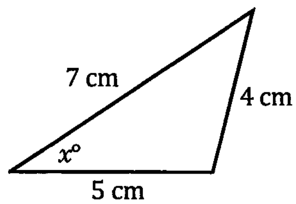
41. A mathematics multiple choice test consisted of 25 questions. Candidates were awarded 4 marks for each correct answer, they lost 3 marks for each incorrect answer but there was no penalty for any questions that were left unattempted. David attempted 23 of the questions and scored 64 marks altogether. How many did he answer correctly?
    Suppose instead that whilst 4 marks were still awarded for each correct answer and 3 were lost for each incorrect answer, 3 marks were also lost for each question not attempted. How many questions must be correctly answered for a mark of at least 50?

Miscellaneous Exercise Twelve.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary work section at the beginning of the book.

1. List all the errors evident in each of the following "solutions".

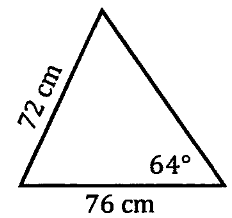
(a



By the cosine rule:

$$\begin{aligned} 4^2 &= 5^2 + 7^2 - 2 \times 5 \times 7 \cos x \\ 16 &= 25 + 49 - 70 \cos x \\ 16 &= 74 - 70 \cos x \\ 16 &= 4 \cos x \\ \cos x &= 0{\cdot}25 \\ x &= 75{\cdot}52248781 \end{aligned}$$

(b



Using Area $= \dfrac{ab \sin C}{2}$

$$\begin{aligned} \text{Area} &= \frac{76 \times 72 \times \sin 64°}{2} \\ &= 38 \times 36 \times \sin 32° \\ &\approx 725 \text{ cm} \end{aligned}$$

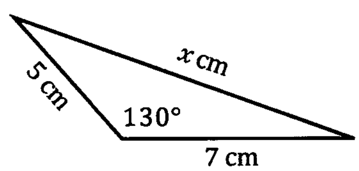
(c) In ΔABC, AB = 6·8 cm,
BC = 7·1 cm
and $\angle$BAC = 65°.
To find the size of $\angle$BCA:

By the cosine rule:

$$\begin{aligned} \frac{6{\cdot}8}{\sin \angle BCA} &= \frac{7{\cdot}1}{\sin 65°} \\ \sin \angle BCA &= \frac{7{\cdot}1 \sin 65°}{6{\cdot}8} \\ &\approx 0{\cdot}9463 \\ \angle BCA &= 71° \text{ or } 109° \end{aligned}$$

to the nearest degree.

(d)



By the cosine rule:

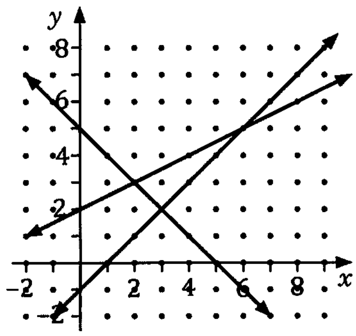
$$\begin{aligned} x^2 &= 5^2 + 7^2 + 2 \times 5 \times 7 \cos 130° \\ &= 25 + 49 + 70 \cos 130° \\ &= 74 + 70 \cos 130° \\ &= 144 \cos 130° \\ &\approx 92{\cdot}5614 \\ x &\approx 9{\cdot}6 \text{ cm} \end{aligned}$$

2. For each of the following classify the variable as one of the following four types:

Nominal categorical	Ordinal categorical	Discrete numerical	Continuous numerical

(a) Nationality.
(b) Height.
(c) Enthusiasm (High, Medium, Low).
(d) Number of people in family.
(e) Distance from home to work.
(f) Time for 400 metres.
(g) Number of people in a marathon.
(h) Gender.
(i) State of Australia a person's main residence is in.
(j) Number of press ups completed in one minute.

3. Using only the graph shown on the right, determine the solutions to each of the following pairs of simultaneous equations.



(a) $\begin{cases} y = x - 1 \\ y = 0{\cdot}5x + 2 \end{cases}$

(b) $\begin{cases} y = x - 1 \\ y = -x + 5 \end{cases}$

(c) $\begin{cases} y = 0{\cdot}5x + 2 \\ y = -x + 5 \end{cases}$

4. Solve the following equations.

(a) $5x - 7 = 11$ (b) $3(2x - 5) + 6 = 40 - x$ (c) $\dfrac{x+3}{2} - 8 = 1$

(d) $\begin{cases} 5x - 3y = 46 \\ x + 2y = 17 \end{cases}$ (e) $\dfrac{x}{7} = \sin 30°$ (f) $\dfrac{7}{x} = \sin 30°$

5. The display on the right shows the lines

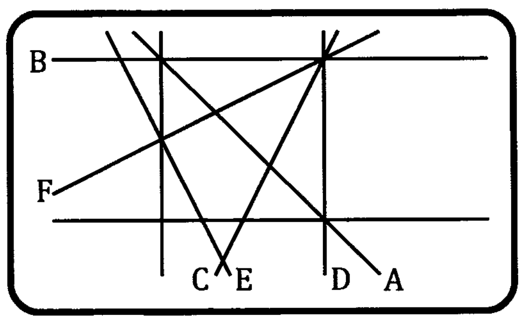
$x = 60$ $y = 60$

$y = 2x - 60$ $y = 0{\cdot}5x + 30$

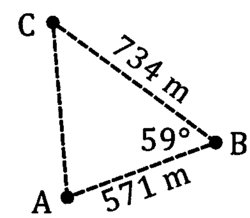
$y = -x + 60$ $y = -2x + 30$

labelled A to F.

Allocate the correct rule to each line.



6. For the situation shown on the right how much shorter is the direct journey from A to C than the journey from A to C via B.



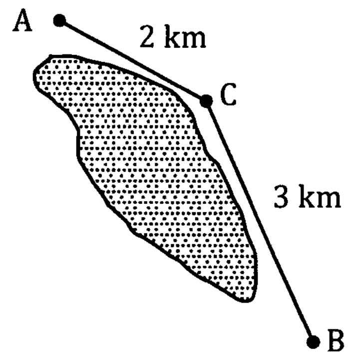
7. A particular "family" of straight lines are related by the fact that they all have equations of the form $y = 3x + c$, each member having a different value for c. What feature do the graphs of all members of this family have in common?

8. A particular "family" of straight lines are related by the fact that they all have equations of the form $y = mx - 7$, each member having a different value for m. What feature do the graphs of all members of this family have in common?

9. A particular "family" of straight lines are related by the fact that they all have equations of the form $x + 2y = c$, each member having a different value for c. What feature do the graphs of all members of this family have in common?

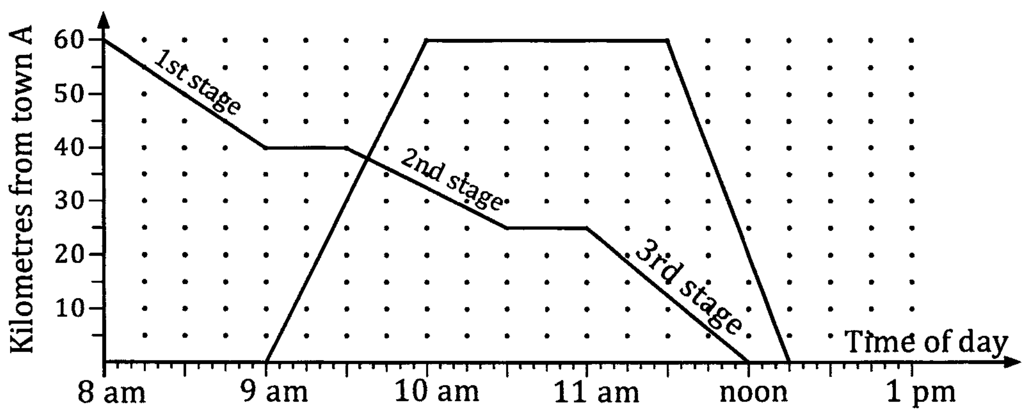
10. From Lookout N^{o}·1 a fire is spotted on a bearing 050°. From Lookout N^{o}·2 the fire is seen on a bearing 020°. Lookout N^{o}·2 is 10 km from Lookout N^{o}·1 on a bearing 120°. Assuming that the fire and the two lookouts are all on the same horizontal level find how far the fire is from each lookout.

11. The road from A to B consists of two straight sections AC, length 2 km, and CB, length 3 km (see diagram). The bearing of C from A is 108° and the bearing of B from A is 132°.



How much further is the road route from A to B than the straight line distance AB?

12. Two towns A and B are 60 km apart and are separated by a long road that can be assumed straight. A cyclist sets off from town B at 8 a.m. one morning and travels to town A in three stages, maintaining an approximately constant speed over each stage and resting for half an hour between stages. A delivery van sets off from town A, travels to town B at an approximately constant speed, stays in B for unloading etc, and then returns to A, again maintaining an approximately constant speed. The distance - time graph of this situation is shown below.



(a) Without doing any calculations it is possible to tell from the graph on which of the three stages the cyclist maintained the greatest average speed. Explain.

(b) What was the cyclist's average speed over each of the three stages?

(c) What was the delivery van's average speed from town A to town B?

(d) What was the delivery van's average speed from town B to town A?

(e) Without leaving town B earlier than it did what average speed would the delivery van have needed to travel back to A at if it was to arrive back at town A before the cyclist reached there?

13. An investor has \$50 000 to invest for one year. She decides to put some of it in a secure deposit account and the rest in a more risky investment. At the end of the year the deposit account pays interest of 8%, the more risky investment pays 18% and the investor receives a total of \$5 600 in interest from these two sources. How much of the \$50 000 had the investor put in the secure deposit account and how much in the more risky investment?

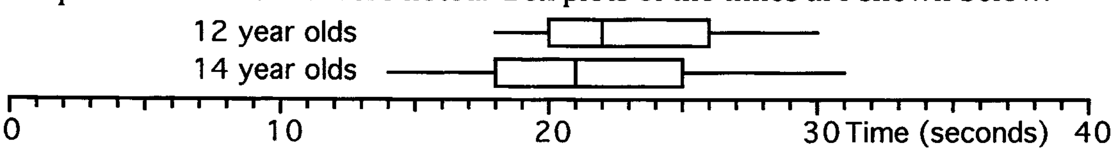
14. The investor of question 13 decides to re-invest her money for a second year. She adds the interest to her original \$50 000 so she now has \$55 600 to invest. She still opts to keep all of this money invested with either the secure deposit account or the more risky investment but she changes the balance of her portfolio. At the end of this second year her total interest is \$4 360 with the deposit account paying 10% and the more risky investment paying 7%. How much of the \$55 600 had the investor put in the deposit account and how much in the more risky investment?

15. Triangle ABC has $a = 5$ cm, $b = 7$ cm and $c = 6$ cm.
    (a) Use the cosine rule to determine the size of $\angle$C.
    (b) Use $\frac{1}{2}ab \sin C$ to determine the area of ΔABC.
    (c) When given the lengths of the sides of a triangle an alternative way of determining the area is to use or **Heron's "s" formula**:

    Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$.

    Calculate the area of ΔABC using this formula.

16. The times taken for some 12 year old students and some 14 year old students to run a particular distance were noted. Box plots of the times are shown below.



Each of the following statements are either incorrect or their correctness cannot be concluded from the boxplot.

(a) Write a few sentences about each of the following statements.
   (i) *For the 12 year old results the box plot extends further to the right of the median than to the left. This shows there are more results involved to the right of the median than there are to the left.*
   (ii) *More 14 year olds were involved than 12 year olds.*
   (iii) *The interquartile range for the 14 year olds was much bigger than the interquartile range for the 12 year olds.*

(b) Write at least five statements that in some way compare the times of the 12 year olds to those of the 14 year olds, with the first two of your five statements being completed versions of the following:
   (i) *The range of times for the 14 year olds (___ seconds) exceeded the range of times for the 12 year olds (___ seconds).*
   (ii) *The fastest 25% of the 14 year olds were ___ than the fastest of the 12 year olds.*